SLIDING OF A BODY OVER A MELTING SURFACE AT A HIGH VELOCITY

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UDC 531.43:621.89

A mathematical model of thermal processes in the motion of one body over the melting surface of another is considered. A simple analytical solution to the problem, applicable in a wide range of velocities, is obtained.

In a number of fields of modern technology, in which the trend toward intensification of processes is increasing, problems associated with the study of friction between different pairs of materials at a high sliding velocity are becoming ever more urgent [1]. The melting of one part of a contacting pair results in a lubrication effect, reducing the frictional force. A similar picture is observed in the motion of a skate over ice. In [2, 3], problems of this kind are studied from the standpoint of the hydrodynamic theory of lubrication developed by Reynolds [4]. That theory, in particular, enables one to calculate the pressure of a liquid between two surfaces moving relative to each other at a high velocity and between which there is a certain angle (the bearing ability of a wedge). In melting as a result of friction, the liquid layer also has a wedge shape. But this is not a wedge into which liquid is injected, and for a given relative velocity of the surfaces it has the opposite direction to that which occurs in bearings, and its nature is completely different. Here the flow rate of liquid varies along the film rather than its pressure, so the Reynolds theory is not applicable in this case. In [3] the problem was considered under the condition that the film thickness is constant along the contact surface, so that there is no wedge. But the assumption that the surfaces between which the melting process occurs are parallel is a strong simplification of the problem.

In the present paper we suggest a solution to the problem using a model that is not based on the hydrodynamic theory of lubrication. Pressure may not play an important role in the liquid layer; the liquid film is formed and maintained by the heat flux from the film itself and by the heat produced in the process of viscous dissipation of energy.

Over the surface of the stationary body (the counterbody), which is assumed to have a relatively low melting temperature, a rigid body (the slider) moves with acceleration. The temperature of both bodies before the onset of motion was equal to the ambient temperature T_0. Dry friction and intense heat release at the contact surface occur in the initial period of motion. A melt film then develops on the trailing part of the contact surface. At a sufficiently high velocity, at a time t from the onset of motion, the liquid film covers the entire contact surface. By this time, the slider has become heated to a thickness [1] $\delta \approx \sqrt{at}$, and the heat flux in its direction will be negligible compared with the flux toward the melting counterbody.

The heat-conduction equation for the moving medium has the form [5]

$$\left(\frac{\partial}{\partial t} + \mathrm{v}\cdot\mathrm{grad}\right)(\rho c T) = \mathrm{div}\,\lambda\,\mathrm{grad}\,T + \Phi, \qquad (1)$$

where Φ is the dissipation function.

The arrangement of the coordinate axes is shown in Fig. 1. The body moves from left to right with a velocity U. In the coordinate system rigidly associated with the body, the velocity within the liquid film at the boundary with the body is $v_x = 0$, and at the melting boundary at the counterbody it is $v_x = U$. The contact width (along the z axis) is assumed to be fairly large, and boundary effects (along the width) are ignored. We consider the quasi-

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Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 6, pp. 964-968, June, 1991. Original article submitted May 21, 1990.

 0022-0841/91/6006-0740$12.50 © 1991 Plenum Publishing Corporation

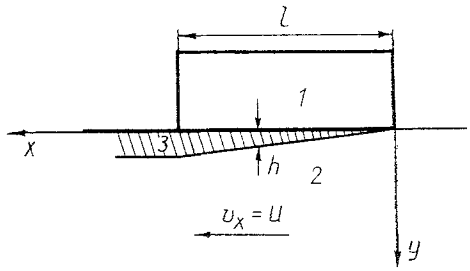

Fig. 1. Diagram of the relative motion of the bodies and arrangement of the coordinate axes: 1) moving body; 2) counterbody; 3) liquid layer.

steady case ($\partial/\partial t = 0$), the liquid is incompressible, and we ignore the temperature dependence of the thermophysical properties of the material. We then write Eq. (1) in the boundary-layer approximation

$$\rho c v_x \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial v_x}{\partial y} \right)^2 . \tag{2}$$

For a small film thickness, a high sliding velocity, and the absence of pronounced pressure gradients, it is natural to assume the velocity profile within the film to be linear,

$$v_x = U \frac{y}{h}, \quad 0 \leqslant y \leqslant h(x), \quad 0 < x \leqslant l, \tag{3}$$

and Eq. (2) then takes the form

$$\rho c U \frac{y}{h} \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial y^2} + \mu \frac{U^2}{h^2} . \tag{4}$$

We write the boundary conditions

$$\left. \frac{\partial T}{\partial y} \right|_{y=0} = 0; \quad T|_{y=h} = T_m . \tag{5}$$

The first condition formulates the assumption that in the quasi-steady state, the heat flux through the contact surface toward the body is negligible. The second condition states that within the counterbody, the temperature of the melt at the liquid–solid interface equals the melting temperature T_m.

If the velocity U of relative motion is not too high (the appropriate criterion will be discussed below), the term of the equation that describes heat transfer will be small and can be omitted. A solution of the simpler equation

$$\lambda \frac{\partial^2 T}{\partial y^2} + \mu \frac{U^2}{y^2} = 0 \tag{6}$$

with the boundary conditions (5) is the expression

$$T(x, y) = T_m + T^* \left[1 - \frac{y^2}{h^2(x)} \right], \tag{7}$$

where

$$T^* = \frac{\mu U^2}{2\lambda} \tag{8}$$

is a quantity that does not depend on x.

If the convective term cannot be neglected, then its influence, as seen from the left side of Eq. (4), is manifested predominantly in the vicinity of $y \simeq h$. We shall assume that convective heat transfer into adjacent hotter layers does not alter the quadratic character of the y dependence of the temperature. Only the thickness h(x) of the liquid film varies; it can be calculated by methods of boundary-layer theory. The application of these methods to our melting problem is even better justified than in ordinary hydrodynamics, since the melting boundaries are physically defined, in contrast, for example, to a viscous boundary layer, the boundary of which is very arbitrary.

We calculate the derivative

$$\frac{\partial T}{\partial x} = 2T^* \frac{y^2}{h^3} \frac{dh}{dx}$$

and substitute it into the left side of Eq. (4):

$$\rho c U T^* 2 \frac{y^3}{h^4} \frac{dh}{dx} = \lambda \frac{\partial^2 T}{\partial y^2} + \mu \frac{U^2}{h^2}.$$

We integrate both sides of the latter equation over y from 0 to h, with allowance for the boundary conditions (5):

$$\rho c U T^* \frac{1}{2} \frac{dh}{dx} = \lambda \frac{\partial T}{\partial y}\bigg|_{y=h} + \mu \frac{U^2}{h}. \tag{9}$$

The heat flux across the liquid layer causes heating and melting of the counterbody, as a result of which the liquid layer widens:

$$-\lambda \frac{\partial T}{\partial y}\bigg|_{y=h} = U\rho\,[r + c(T_m - T_0)]\,\frac{dh}{dx}. \tag{10}$$

Substituting the latter equation into (9) and grouping terms, we have

$$\frac{dh}{dx}\left[\frac{1}{2}\rho c T^* U + \rho U r + \rho U c (T_m - T_0)\right] = \mu \frac{U^2}{h} \tag{11}$$

or

$$\frac{dh^2}{dx} = 2\nu U/[r + c(T_m - T_0 + T^*/2)]. \tag{12}$$

Taking $h|_{x=0} = h_0 \simeq 0$ and integrating, we obtain

$$h = \left[\frac{2\nu U x}{r + c(T_m - T_0 + T^*/2)}\right]^{1/2} \tag{13}$$

or

$$h = \left(\frac{2\nu U/c}{r/c + T_m - T_0 + \mu U^2/4\lambda}\, x\right)^{1/2}. \tag{14}$$

If the thickness of the liquid film between the surfaces is known, it is easy to calculate the frictional force. The drag force acting on an elementary area is

$$dF = \mu \frac{U}{h(x)}\, dx dz.$$

The total frictional force acting on the surface of a rectangular body with a size $\ell \times b$ is

$$F = \int_0^b dz \int_0^l \frac{\mu U}{h(x)}\, dx = (2Ulb^2\rho\mu)^{1/2}\,[r + c(T_m - T_0 + \mu U^2/4\lambda)]^{1/2}. \tag{15}$$

Let us estimate the range of relative velocities in which Eqs. (14) and (15) can be valid.

The thickness of the liquid film decreases with decreasing velocity. But here there is a limit beyond which the hydrodynamic description itself loses meaning. In the small vicinity of the point x = 0, where the leading end of the moving body lies, the minimum thickness of the liquid film is h_0. Here the initial equation (2) no longer provides an adequate description, so the subsequent equations have an estimative nature. Let us turn first to Eqs. (9) and (10). In the region of interest to us, we have $\Delta x \approx \Delta h \approx h_0$, from which we have

$$\mu \frac{U^2}{h_0} = U\rho \, [r + c(T_m - T_0)]. \tag{16}$$

If we consider that in this region, the dissipation function Φ should be represented by a number of other terms in addition to the second term on the right side of Eq. (2),

$$\Phi = \mu \left[\left(\frac{\partial v_x}{\partial y} \right)^2 + \left(\frac{\partial v_y}{\partial x} \right)^2 + 2\, \frac{\partial v_x}{\partial y} \frac{\partial v_y}{\partial x} \right],$$

then the left side of Eq. (16), which determines the "input" side of the energy balance in the vicinity of the point x = 0, will be four times larger, and for the corresponding limiting velocity we have the equation

$$U_1 = \frac{h_0}{4\nu} [r + c(T_m - T_0)]. \tag{17}$$

The limiting velocity is related to the limiting thickness h_0 of the liquid film at the front end of the sliding body, which cannot, in any case, be less than the molecular size, 1 Å, for example. There is some arbitrariness in the choice of the parameter h_0, but we can expect that its numerical value will not depend on the materials of the contacting pair or on the shape and size of the roughness elements of the moving body near its end.

With increasing velocity, the thickness of the liquid film increases, but only to a certain limit, as seen from Eq. (14), after which it may decrease again. The limit is reached at velocities for which the contribution of convection to the energy balance equals the contribution of heat conduction:

$$U_2 = 2 \left(\frac{r/c + T_m - T_0}{\mu/\lambda} \right)^{1/2}. \tag{18}$$

But if the contribution of the convective term in Eq. (4) is large, the initial assumption of a quadratic temperature profile obviously becomes false, so that we must have $U < U_2$.

Let us give some numerical results.

In the motion of a body over an ice surface, if $T_m - T_0 = 0$ and $h_0 = 4$ Å, estimates give $U_1 = 19$ m/sec and $U_2 = 312$ m/sec. At a velocity $U = 200$ m/sec and for $\ell = b = 0.1$ m, the temperature of the body at the contact surface will exceed the melting temperature of ice by $T^* = 65$ K, the maximum thickness of the liquid film will be $h = 12.3 \cdot 10^{-6}$ m, and the frictional force will be F = 580 N.

If the body moves over a tin surface, then for $T_m - T_0 = 232$ K and $h_0 = 4$ Å we will have $U_1 = 43$ m/sec and $U_2 = 5.5 \cdot 10^{-3}$ m/sec. Let the velocity be U = 200 m/sec and $\ell = b =$ 0.1 m. We then have $T^* = 1.1$ K, $h = 9.6 \cdot 10^{-6}$ m, and F = 800 N.

In conclusion, let us formulate our results. We obtained a simple analytical solution to the problem of the sliding of a body at a high velocity over a melting surface which is applicable in a wide velocity range. The thickness of the liquid film as a function of the coordinate and velocity was determined. The intensity of heat release in the film and its thickness depend on the distance to the front end of the moving body, but the temperature over the entire contact surface is constant and exceeds the melting temperature by an amount proportional to the square of the relative velocity.

NOTATION

t, time; T, temperature; T_0, ambient temperature; T_m, melting temperature; v, velocity of a point in the liquid medium; U, relative velocity of the bodies; ρ, liquid density, kg/m^3; c, specific heat of the liquid, $J/(m^3 \cdot K)$; λ, thermal conductivity coefficient of the liquid, $W/(m \cdot K)$; μ, dynamic viscosity, kg/(m·sec); $\nu = \mu/\rho$, kinematic viscosity, m^2/sec; r, specific heat of melting, J/kg; h, thickness of the liquid film; a, thermal diffusivity coefficient of the material of the moving body; ℓ, length of the sliding body; b, width of the body.

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TESTING TWO-TEMPERATURE THERMAL-CONDUCTION THEORY FOR CARBON ROD COMPOSITES

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UDC 536.2.01

Measurements have been used to test software for calculating the temperature pattern in a reinforced medium in the two-temperature approximation and for determining model parameters.

Rod composites are widely used, which requires models that adequately reflect heat transfer there; the usual approach is based on homogenizing the composite via effective thermophysical characteristics [1]. The errors are very much dependent on obedience to the conditions for equivalence between a homogeneous medium and the initial heterogeneous one [2], which complicates determining the effective thermophysical parameters. The effective thermal conductivity of a heterogeneous material in general is dependent on time [3].

An alternative description is the two-temperature conduction model, which avoids those difficulties. A representative elementary volume is distinguished, which contains one reinforcing rod in the matrix, and for which one writes averaged conduction equations for each component together with Henry's equation, which relates the heat fluxes between the components q_{ij} to the mean temperatures:

$$q_{ij} = \alpha_c(\hat{T}_i - \hat{T}_j). \quad (1)$$

The correctness of the model has been discussed [4, 5]; it has been used in model treatments [6, 7]. However, its use is hindered by the lack of data on the thermophysical characteristics of the components and also α_c.

Here we present a model for that approach and measurements on the thermal conductivities of carbon rods and matrix; temperature patterns as calculated from the approach are compared with experiment.

Model. We consider a material in which the rods can be divided into two groups: ones in the x–y planes parallel to the surface (denoted by f_{x-y}) and ones parallel to the z axis, f_z; the spaces between the rods are filled with matrix m. Partial homogenization is performed [8] for the f_{x-y} rods and m, and we convert from the multicomponent medium to a two-component one.

Central Special Engineering Research Institute, Khot'kovo. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 6, pp. 968-974, June, 1991. Original article submitted June 26, 1990.

0022-0841/91/6006-0744$12.50

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